Optimization of the Cellular Beams by Applying Particle Swarm Algorithm (PSO)

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ABSTRACT
Structural optimization for the purpose of reducing the costs of designing and implementation has always been of interest to engineers. The Cellular Beams Production due to higher strength and lower cost has been considered in recent years. Among the fundamental characteristics of these beams can be implied to architectural features, strength and the flexural stiffness of the beams without the need to increase the weight of the beams. In the present paper, the overall cost is considered as the major function and the effect of welding and cutting parameters have also been assessed to design the beams during the optimization. The required boundaries are also considered in designing beams by regulations code, (AISC-ASD). Eventually, leveraging variables and design constraints, optimization of beams by the use of Particle Swarm Optimization Algorithm has been illustrated here.

Keywords: optimization, Particle Swarm Optimization Algorithm (PSO), cellular beams

INTRODUCTION
To provide a structure with resistance ratio over much loading, beams with open sections has been taken into account by engineers and users in recent decades. Two common types of these beams are beams with hexagonal holes which are known as castellated beams and beams with circular holes which are known as cellular beams. The preliminary reasons of the development of cellular beams can be attributed to their architectural characteristics. In general, some of the advantages of these beams are:
1. The greater height over the main beam which eventually results in flexural strength and flexural rigidity.
2. The architectural characteristics of these beams which are useful for electrical installations and plumbing.
3. The main advantage of these beams is that it augments bending load capacity without increasing the weight of the beam, which eventually leads to a cost-effective structure.

Due to the aforementioned advantages of these beams, today, these beams are used in many large projects such as sports stadiums and public places and tall buildings and parking lots. Optimization as a combination of mathematics and economic issues has been applied in various disciplines like engineering in recent years. In general, optimization techniques can be divided into two major categories: heuristics and meta-heuristics methods. Heuristics methods have concentrated upon combination of mathematical planning. Types of mathematical planning are linear programming, nonlinear programming and dynamic programming. It has a high convergence speed and solution offered by them have a high degree of accuracy, but their major problem lies in choosing initial search point and local optimum. Therefore, in recent decades, the meta-heuristic methods inspired by natural phenomena have been considered. Among them are Forbidden Methods Search (TS), Ant Colony Optimization Algorithm (ACO), Simulated Annealing Method and Particle Swarm Optimization (PSO). The principal function of these techniques lies in overall optimizing. Thus, with respect to the widespread use of cellular beams, the optimization of these beams for the purpose of reducing costs by using Particle Swarm Algorithm is taken into account in this study.

2. Particle Swarm Algorithm
Modern Particle Swarm Algorithm has been put forward by inspired from the movement of birds by Eberhard and Kennedy in 1995. Eberhard and Kennedy surveyed the application of Particle Swarm in the development of artificial neural networks. With respect to the previous studies about the need of every particle for recognizing its and companions
experiences, they introduced the gbest models for flight experience of companions throughout the 
community and Ibest model for the experience of flight attendants in the local neighborhood. They 
developed equations based on 3 principles: Each particle is absorbed into the nest location. Each 
particle remembers its position closest to nest. Each particle shares its information with all other 
particles adjacent to the nest about the nearest place. Therefore, the two equations represented by 
them which are dominant over changes in the designing variables are as follows:
\[ V_{i}^{t+1} = V_{i}^{t} + C_{1} \times \text{rand}1_{i} x (P_{best}^{t} - x_{i}^{t}) + C_{2} \times \text{rand}2_{i} (g_{best}^{t} - x_{i}^{t}) \] (1)
\[ x_{i}^{t+1} = x_{i}^{t} + V_{i}^{t+1} \] (2)
The first equation represents updating the velocity of the particle and the second equation represents 
updating the position of the particle.

As you see, the first equation consists of 3 statements:

- The first sentence which is indicator of current speed and pay its attention to the 
direction of particle’s movement.
- The second sentence for the purpose of coordinating the search, pay its attention to 
the best position of the particle.
- The third sentence for the purpose of coordinating the search, pay its attention to 
the best situation experienced by the all particles.

In the two major links provided by the method of Particle Swarm, \( V_{i}^{t} \) and \( x_{i}^{t} \), respectively, indicate the 
current speed of the particle and its current position, and \( i \) represents a design variable and \( t \) is the iteration 
number. \( \text{rand}1_{i} \) and \( \text{rand}2_{i} \) are two random number which 
are selected between \([0, 1]\).

\( C_{1} \) and \( C_{2} \) which are sometimes shown as \( q_{1} \) and \( q_{2} \) are the acceleration coefficients and have also 
known as personal and social learning factors. \( P_{best}^{t} \) indicates the particle’s best position to \( t \) stage 
and \( g_{best}^{t} \) is the best position experienced by the particles to \( t \) stage.

3. Design criteria

Before investigating the design criteria of cellular beams, it must be mentioned that the regulations 
applied in this study is (AISC) in terms of Allowable Tension Method.

3.1 Geometric criteria

For the purpose of providing cellular beams from the 
main beam, a shear is done as a semicircular form 
across the central line and then the 2 sections which 
were cut are placed next to each other as a result of 
transfer and are welded together. (Figure 2)
The holes diameter and the distance between them 
can be variable, but some restrictions applied about 
them should be considered here [4].
\[ 0.08 \leq \frac{D}{D} \leq 0.6 \] (3)
\[ 1.25 \leq \frac{H_{c}}{D} \leq 1.75 \] (4)

3.2 Bending criteria

Fig. 1. Production of cellular beams

One of the main criteria in the analysis and design of cellular beams is bending criteria.
Bending stress created by these beams consist of 2 parts:
1) Axial stress due to bending of \( M_{x} \)which possess 
linear distribution in \( H_{c} \) depth.
2) Axial pressures due to secondary bending with a 
linear distribution in depth \( d_{P} \).
Therefore, it can be concluded that the total stress of 
main bending in ending Tars and the Tar above the 
cellular stress and axial stress of secondary bending 
in the ending Tars of T-shaped section and in the 
critical area of cellular beams which is on the basis of
\( \theta \)’s variable angle, should be less than the allowable amount of regulations. 
\[
f_i = \left| \frac{M_x}{S_1} \right| + \left| \frac{m_x}{I_{st}} \right|
\]  
(5)

\[ f_x = \left| \frac{M_x}{S_2} \right| + \left| \frac{m_x}{I_{sb}} \right| \]

where \( M_x \) is the primary bending and \( m_x \) is the secondary bending in the section.

3.3. Shear measurement

- Criteria for vertical shear
On the hole, a cut in the form of a shield section provides a vertical shear stress, and its distribution is paraboloid and its ratio is as follows:

\[ (f_{\text{v}})_{\text{max}} = \frac{v_x q_0}{2 l_c t_w} \leq F_{\text{v}} \]

(7)

\( Q \) is the maximum area moment and \( l_c \) is the maximum moment of inertia of shear form section and \( f_{\text{v}} \) is the maximum allowable tension in regulations.

- Measuring horizontal shear
If we take a step of hole into account, it becomes clear that due to the change in axial forces, a shear force is created among the holes and its ratio is as follows:

\[ v_h = q s \]

(8)

Where \( q \) is the horizontal shear flow at the axis of bending beam, and since the distribution of shear stress in the web in the distance between the holes is with a parabolic distribution, therefore, it is maximum value is:

\[ (f_{\text{h}})_{\text{max}} = 1.5 \frac{v_h}{e t_w} \leq F_{\text{h}} \]

(9)

Generally, if the values of shear stress (vertical or horizontal) exceed the permissible values, we can meet this deficiency by filling the area between the holes in this problem.

3.4 Compressive Stress Criterion in Web

Fig. 2. The effect of main and secondary beams on cellular beams

Forces applied on cellular beams whether in widespread level or concentrated into a beam, whenever they lie in upper wing, half of the above loads must pass through perforated area of web in web’s section. In this manner, web at the height of \( D \) acts as a compressive column, therefore, compressive columns stress is as follows:

\[ f_a = \frac{w_s / 2 + p / 2}{e t_w} \leq F(a) \]

(10)

Where \( F(a) \) is the compressive allowable stress and is determined by the column slender ratio.

3.5 Deformation Criterion
Deformation of cellular beams are generally made up of 2 parts.
1) Deformation of pure bending is obtained from simple bending theory.
2) Deformation of shear. This additional deformation makes behavior of beam, like a Virendel beam.

According to conducted researches, we can write:

\[ Y = C_1 L^3 + C_2 L \]

(11)

The first sentence \((C_1 L^3)\) represents the deformation caused by the bending and the second sentence \((C_2 L)\) represents the shear deformation of the section. \( C_1 \) and \( C_2 \) coefficients are obtained by curve’s fitness techniques and on the basis of load kind and support conditions or if time is centralized, it can be said

\[ Y = K_1 \frac{p l^3}{E I_{\text{eff}}} + K_2 \frac{p l}{E A_{\text{eff}}} \]

(12)

\( K_1 \) and \( K_2 \) are determined on the basis of the anchor’s specified criteria and \( A_{\text{eff}} \) and \( I_{\text{eff}} \) are respectively the effective moment of inertia and effective cutting area.
4. Objective function
Optimization problems in civil engineering structures are mainly the aim of optimizing the weight of these arrows, but while on these bars the issue of cutting and welding are being put forward, so the best option is to select the target cost. In the present research, the objective function is determined as follows:
\[
\text{cost} = C_1 L_1 + C_2 L_2 + C_3 \rho \Delta L - C_4 \rho \Delta L
\]  
(13)  
\(L_1\) is cutting length and \(L_2\) is welding length and \(L\) is the length of the beam and \(\Delta\) is the cross section of main beam and \(A\) is the omitted cross-section (waste), and \(C_1 = 0.3\) and \(C_2 = 1\). Cost coefficients in terms of length scale is determined as \$\) dollars for welding and cutting and \(C_3 = 0.8\) and \(C_4 = 0.2\), respectively. indicate the consumption on the main beam and steel scrap determined in dollars per unit weight.

5. Design variables
At first, designing cellular beams requires selecting the section of main beam (W-Section) in order to produce castellated beams with circular holes. On the other hand, in castellated beams with circular holes, diameter selection and distance between them are design variables. (Because no one else is available)

Therefore, in optimal designing of castellated beams with circular holes, the purpose is to find the vector design as follows:
\[
[I] = [I_1, I_2, I_3]^T
\]  
(14)  
\(I_1\) is the desired section number and \(I_2\) is the diameter of holes by considering the applied restrictions and \(I_3\) is the correct number of holes (distance between them).

6. Design constraints
In general, the cellular beams are influenced by a large number of geometric constraints and design limitations, on the basis of diameter and the distance between the holes, we can write:
\[
g_1 = 1.08 D - S \leq 0
\]  
(15)  
\(g_2 = S - 1.6 D \leq 0\)
(16)  
\(g_3 = 1.25 D - H \leq 0\)
(17)  
\(g_4 = H - 1.75 D \leq 0\)
(18)
And in terms of design criteria:
\[
g_5 = f_1 - F_{p1} \leq 0
\]  
(19)  
\(g_6 = f_2 - F_{p2} \leq 0\)
(20)
\(F_{p1}\) and \(F_{p2}\) are the allowable amounts of regulation for the impacts of total axial stress due to bending of the primary and secondary levels:
\[
g_7 = (f_{uv})_{\text{max}} - F_{u} \leq 0
\]  
(21)  
\(g_8 = (f_{rh})_{\text{max}} - F_{h} \leq 0\)
(22)
\(F_u\) is the allowable vertical shear stress of regulations and \(F_h\) is the maximum amount considered as allowable horizontal shear stress in accordance with regulations.
\[
g_9 = f_a - F_{(a)} \leq 0
\]  
(23)  
\(F(a)\) is the maximum compressive stress in the column.
\[
g_{10} = y_{\text{max}} - L/360 \leq 0
\]  
(24)  
\(L / 360\), is to limit the maximum allowable amount of deformation of the beam.

7. Penalty functions
Various approaches are available for imposing the design constraints, they are most commonly used in engineering problems and are proposed by Rajeev and Kishnamorthy.

It can be said that the penalty function is a measure to determine the amount of each project’s violation from allowable limit. In fact, these are the plans in which design constraints are not observed. This function can be forfeited in order to reduce their authority. In this study, after normalizing the design constraints:
\[
\text{if } g_i \leq 0 \Rightarrow g_i = 0
\]  
\(i = 1, 2, \ldots, n\)
(25)  
After calculating the amount of forfeiting for each deviation of design constraints in the sentence, these forfeits are added to the initial cost function by applying some coefficients. Eventually, the final function can be achieved.
\[
\text{mer} = \text{cost} + \varepsilon_1 (\sum_{i=1}^{n} g_i)^{\varepsilon_2}
\]  
(26)  
\(\varepsilon_1\) and \(\varepsilon_2\) are the coefficients of the fitness function and \(\sum g_i\) is the crime rate.

8. Numerical example
A beam for the length of 4 meters is widely influenced by a dead load of 5000 Newton/ meters and a concentrated force of \(50 \times 10^3\) N is located at midcaps. Modulus of elasticity of steel is considered \(2.05 \times 10^7\) \(\text{N/cm}^2\) and steel yield stress is considered \(2400\) \(\text{kg/cm}^2\). The sections used for designing are varied from \(W 4 \times 13\) to \(W 44 \times 335\) and the hole diameter and the distance between the holes are calculated up to 0.1mm and the number of holes in designing is taken into account by a one unit change.
Fig. 3. Cellular beams under concentrated and extensive loads

By applying zero cost cutting and welding:

Table 1: Results of particle swarm algorithm in the Cellular beam under concentrated and extensive loads

<table>
<thead>
<tr>
<th>Best Results (PSO)</th>
<th>Particle size</th>
<th>Number of iterations</th>
<th>Name of Section</th>
<th>Diameter of hole D (mm)</th>
<th>Number of holes (n)</th>
<th>Marginal Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>200</td>
<td>W12×19</td>
<td>357</td>
<td>9</td>
<td>81.02</td>
</tr>
<tr>
<td>Average results</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>82.39</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.62</td>
</tr>
</tbody>
</table>

Table 2. Results of the removal of cutting cost in the Cellular beam under concentrated and extensive loads

<table>
<thead>
<tr>
<th>Name of Section</th>
<th>Diameter of hole D (mm)</th>
<th>Number of holes (n)</th>
<th>Marginal Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO W12×19</td>
<td>356.9</td>
<td>9</td>
<td>77.16</td>
</tr>
</tbody>
</table>

Table 3. Results of the removal of welding cost in the Cellular beam under concentrated and extensive loads

<table>
<thead>
<tr>
<th>Name of Section</th>
<th>Diameter of hole D (mm)</th>
<th>Number of holes (n)</th>
<th>Marginal Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO W12×19</td>
<td>356.9</td>
<td>9</td>
<td>80.13</td>
</tr>
</tbody>
</table>

Table 4. Results of the removal of cutting and welding cost in the Cellular beam under concentrated and extensive loads

<table>
<thead>
<tr>
<th>Name of Section</th>
<th>Diameter of hole D (mm)</th>
<th>Number of holes (n)</th>
<th>Marginal Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO W12×19</td>
<td>357.2</td>
<td>9</td>
<td>76.35</td>
</tr>
</tbody>
</table>
Fig. 4. The algorithm converges to the optimal solutions in cellular beams under concentrated load at fixed intervals.

9. Conclusion
In the present study, it has been indicated that for optimal designing of beams, design variables are as follows: selection the cross section of main beam, hole diameter, the distance between the holes or hole numbers. On the other hand, due to the impact of cutting and welding cost in the proposed model it has been indicated that cutting parameter has a greater impact than the welding parameter in the process of optimizing and designing these beams. On the other hand, remove or change on any of the above parameters can also change the size or height of castellated beams cutting, and also causes change in the distance between holes and is rarely effective in the number of holes.

References